CFD Elliptic Analysis of Anisotropic Flow in the Wake of a Wind Turbine

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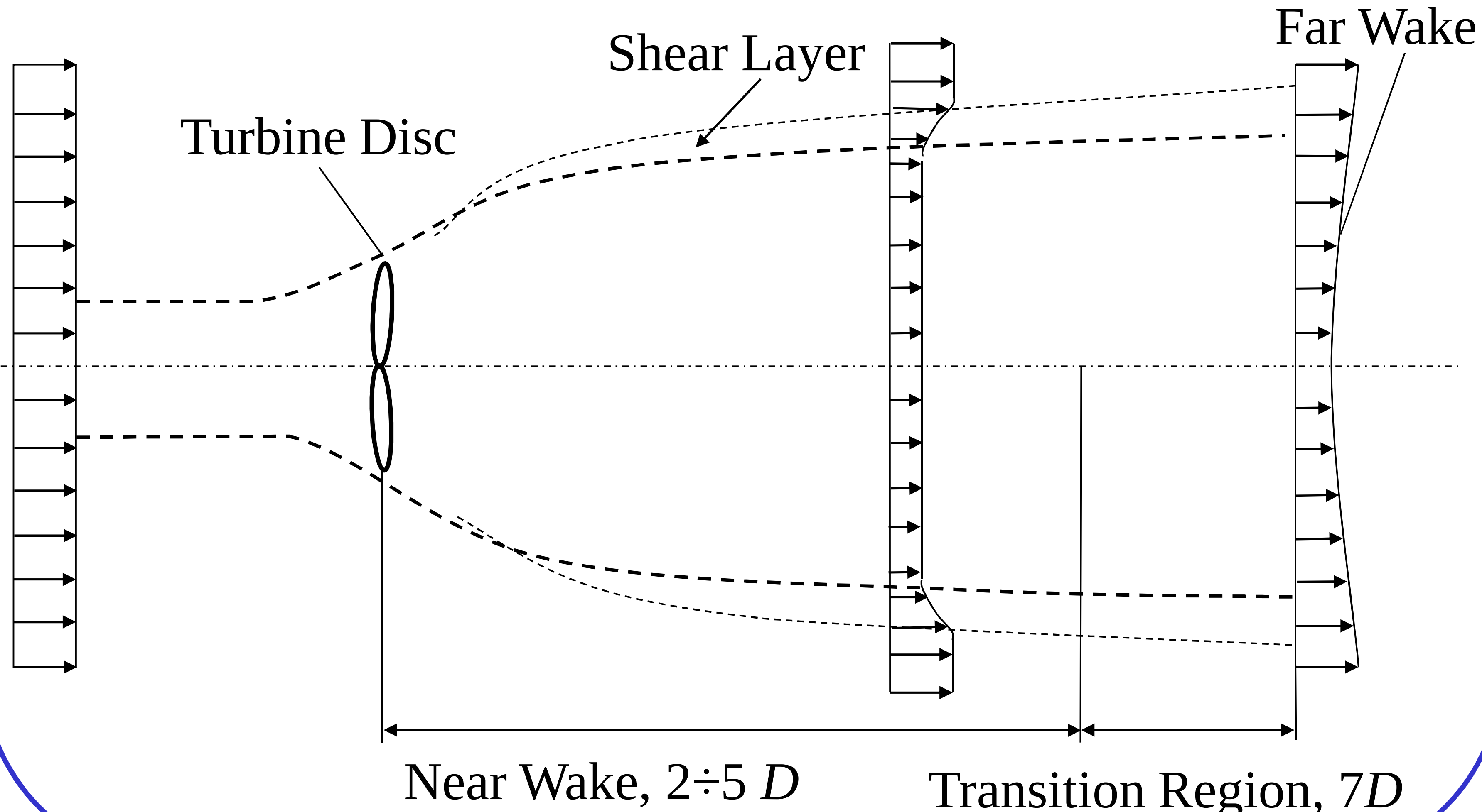
Research Objectives

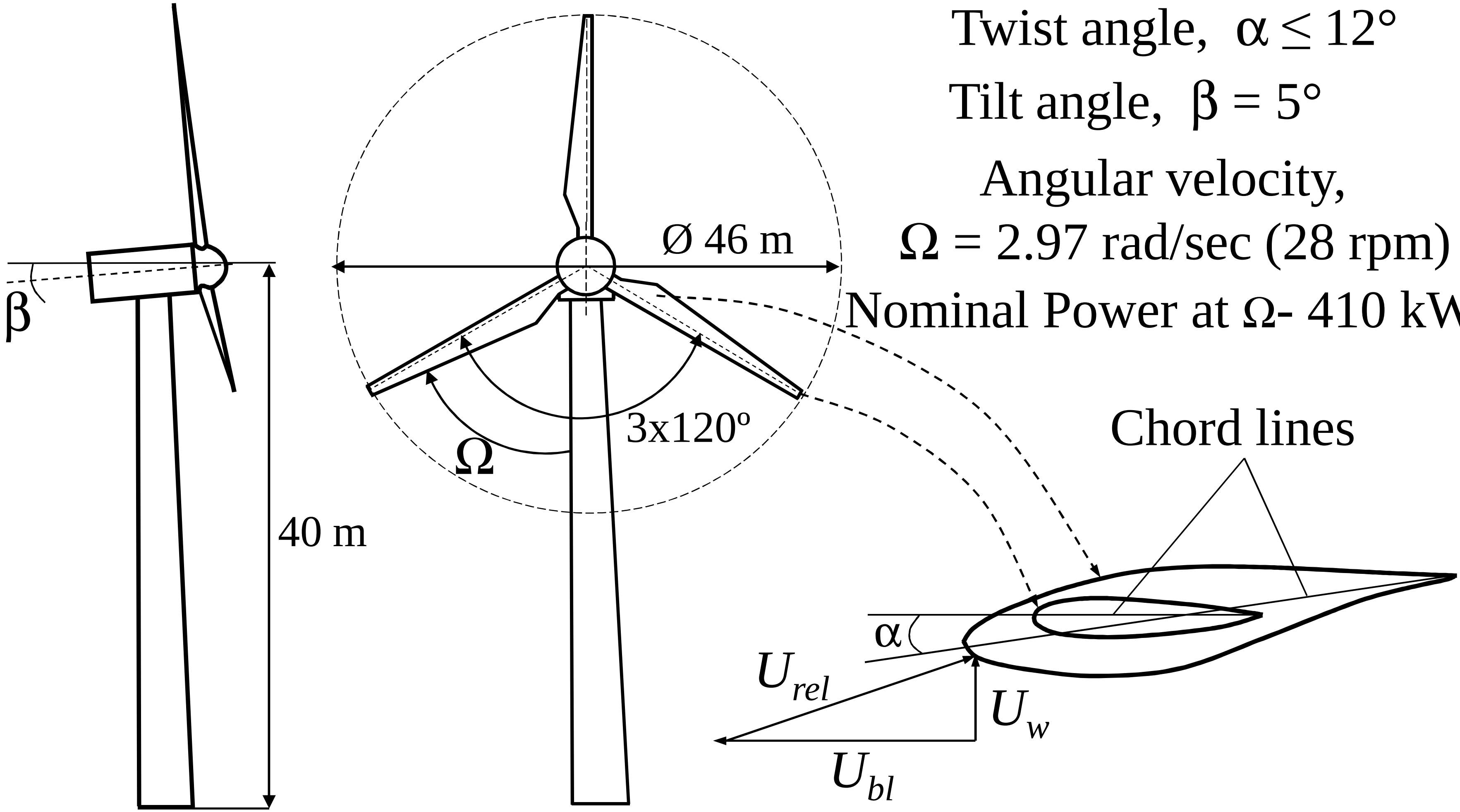
The main objectives of the present work are:

- To overcome stringent limitations imposed in other works in the field.
- to develop a reliable CFD model for a single wind turbine full scale wake analysis.
- to provide a powerful tool for wind turbine engineers as a means for enhancing wind turbine efficiency at the design stage.
- to investigate the potential use of the developed model for the multiple turbine wake analysis.

Schematic Representation of the Wind Turbine Wake



Geometrical Properties of the Wind Turbine



Twist angle, $\alpha \leq 12°$

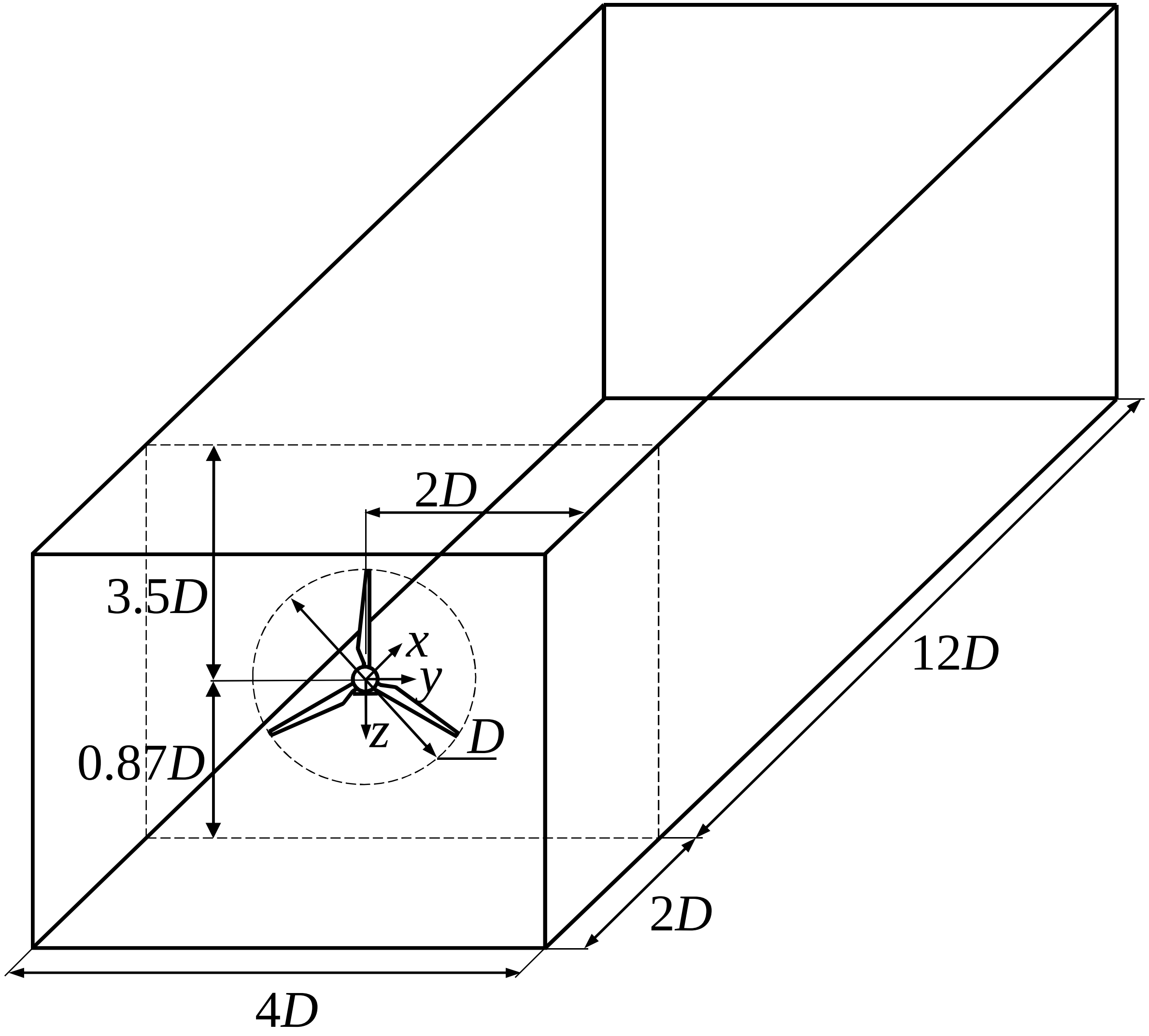
Tilt angle, $\beta = 5°$

Angular velocity,

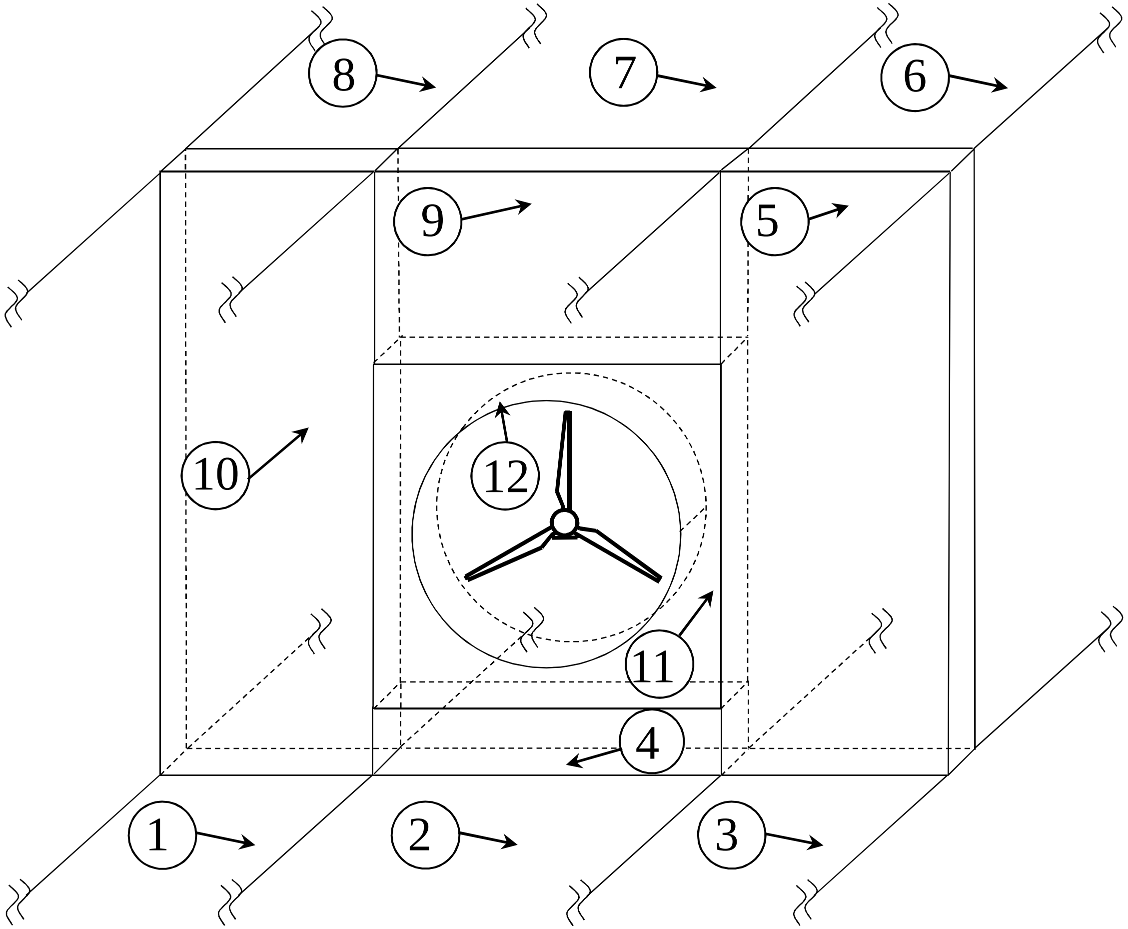
$\Omega = 2.97$ rad/sec (28 rpm)

Nominal Power at Ω- 410 kW

The Computational Domain



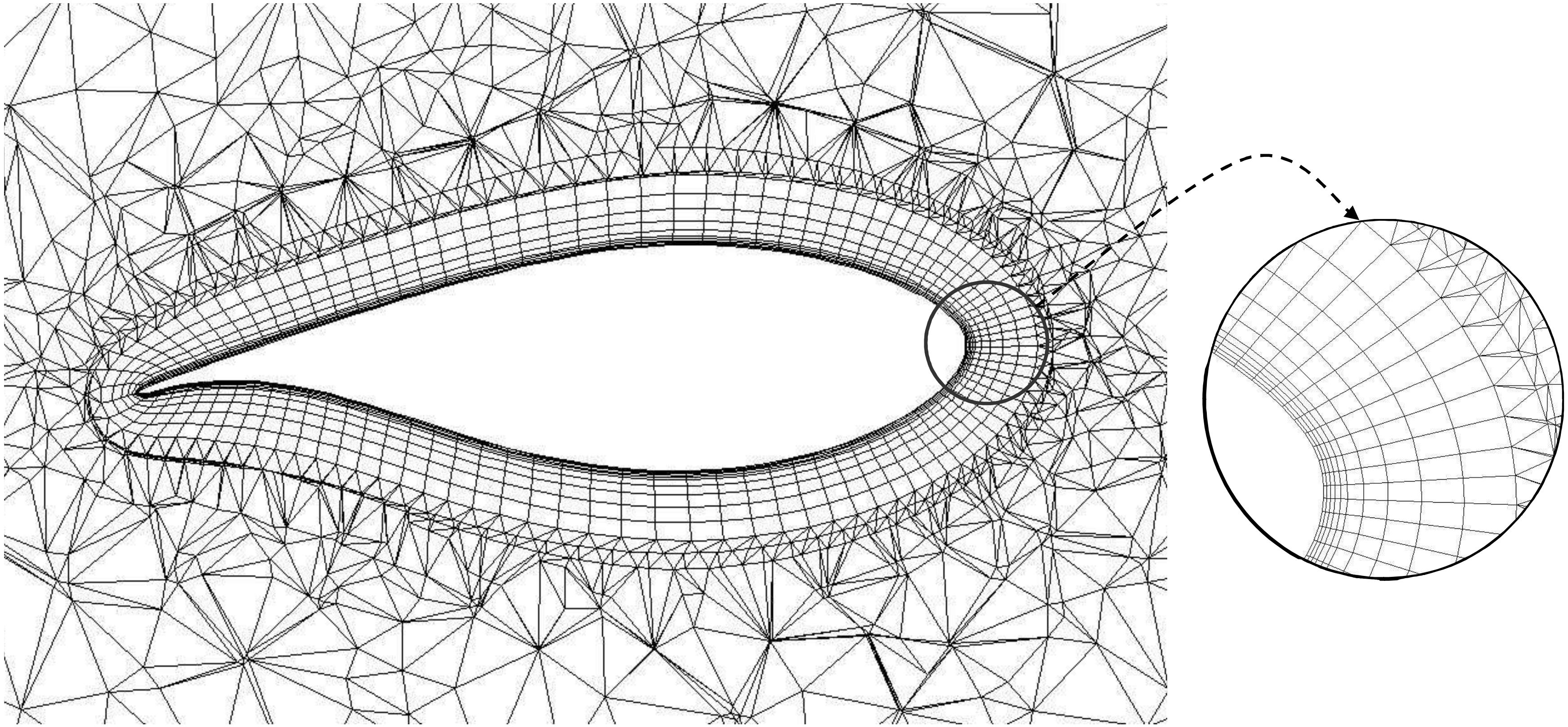
Computational Domain Discretization



Overall cell quantity
~ $5x10^6$

Boundary layer structure consisting of 10 sequentially layers with a first layer thickness (ground) of 100 millimeters and a growth ratio of 1.2 was attached to the domain bottom surface, ($y^+ \in$ [30, 300] -turbulent BL)

The Typical Mesh Structure at the Blade Vicinity



The BL structure consists of 5 sequentially layers characterized by a first layer thickness of 2 millimeters and a growth ratio of 1.2, having an overall thickness of about 15 *mm*. (y^+Є [30, 300]- turbulent BL)

The Basic Model Assumptions

- steady state (moving reference frame) and incompressible flow
- No rotor tilt
- Negligible external forces (gravitation)
- Neutral atmospheric conditions (constant values of inlet velocity and turbulence intensity)
- Reynolds-averaged approach for the turbulence modeling:

$$\mathbf{U} = \mathbf{u} + \mathbf{u}'$$

Governing Equations

The Continuity Equation: $$\frac{\partial u_i}{\partial x_i} = 0$$

The momentum equation:

$$u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\mu}{\rho}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(-\overline{u_i^{'} u_j^{'}}\right)$$

where $\overline{u_i^{'} u_j^{'}}$ are the time averaged products of the fluctuating velocity components, known also as Reynolds stresses.

Governing Equations (Contd. 1)

Exact transport equations for the individual Reynolds stresses (anisotropic turbulence):

$$\underbrace{\frac{\partial}{\partial x_k}\left(u_k \overline{u_i' u_j'}\right)}_{C_{ij} \equiv \text{Convection}} = \underbrace{-\frac{\partial}{\partial x_k}\left[\overline{u_i' u_j' u_k'} + \overline{\frac{p}{\rho}\left(\delta_{kj} u_i' + \delta_{ik} u_j'\right)}\right]}_{D_{T,ij} \equiv \text{Turbulent Diffusion}} + \underbrace{\frac{\partial}{\partial x_k}\left[\frac{\mu}{\rho}\frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)\right]}_{D_{L,ij} \equiv \text{Molecular Diffusion}}$$

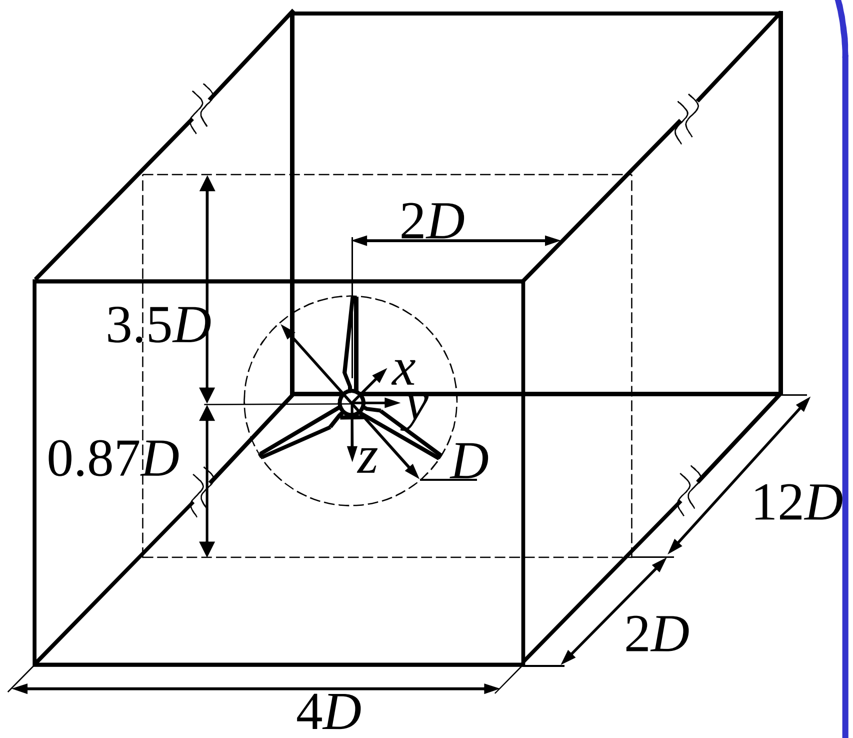
$$\underbrace{-\left(\overline{u_i' u_k'}\frac{\partial u_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial u_i}{\partial x_k}\right)}_{P_{ij} \equiv \text{Stress Production}} + \underbrace{\overline{\frac{p}{\rho}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}}_{\phi_{ij} \equiv \text{PressureStrain}} - \underbrace{2\frac{\mu}{\rho}\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{\varepsilon_{ij} \equiv \text{Dissipation}}$$

$$\underbrace{-2\Omega_k\left(\overline{u_j' u_m'}\varepsilon_{ikm} + \overline{u_i' u_m'}\varepsilon_{jkm}\right)}_{F_{ij} \equiv \text{Production by System Rotation}}$$

Terms to be modeled

Approximate RSM Equations

Boundary conditions



No slip b.c.

bottom: $\boldsymbol{u}\left(x, y, z = 0.87D\right) = 0$

No penetration b.c.

blades: $\Omega(blades) = 2.97\hat{e}_x \ rad/\sec$

Inlet and outlet b.c.

$$u_x\left(x = -2D, y, z\right) = U_0 \quad \text{and} \quad p\left(x = 12D, y, z\right) = p_a$$

Symmetry b.c.

$$u_y\left(x, y = 2D, z\right) = u_y\left(x, y = -2D, z\right) = u_z\left(x, y, z = -3.5D\right) = 0$$

$$\frac{\partial \Phi}{\partial y}\left(x, y = 2D, z\right) = \frac{\partial \Phi}{\partial y}\left(x, y = -2D, z\right) = \frac{\partial \Phi}{\partial z}\left(x, y, z = -3.5D\right) = 0$$

Moving Reference Frame Approach

Moving reference frame approach: $\boldsymbol{u_r} = \boldsymbol{u} - (\boldsymbol{\Omega} \times \boldsymbol{r})$

Transient Effects Inherently Neglected

The Continuity Equation: $\nabla \bullet \boldsymbol{u}_r = 0$

The momentum equation:

$$(\boldsymbol{u}_r \bullet \nabla)\boldsymbol{u}_r + (2\boldsymbol{\Omega} \times \boldsymbol{u}_r + \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{r}) = -1/\rho \nabla p + \mu/\rho \nabla^2 \boldsymbol{u}_r + \nabla \bullet \left(\overline{\boldsymbol{u}'\boldsymbol{u}'}\right)$$

Exact transport equations for the individual Reynolds stresses in terms of $\boldsymbol{u_r}$ are not written for the sake of brevity

The model validation

1. **The turbine power deviation between the CFD model and the real turbine without vortex generators does not exceed 12 %.**

2. **Power- speed tests are not necessarily performed with new blades (surface not hydraulically smooth- possible earlier BL separation).**

3. **An additional analysis was performed only on a downstream computational sub-domain:**

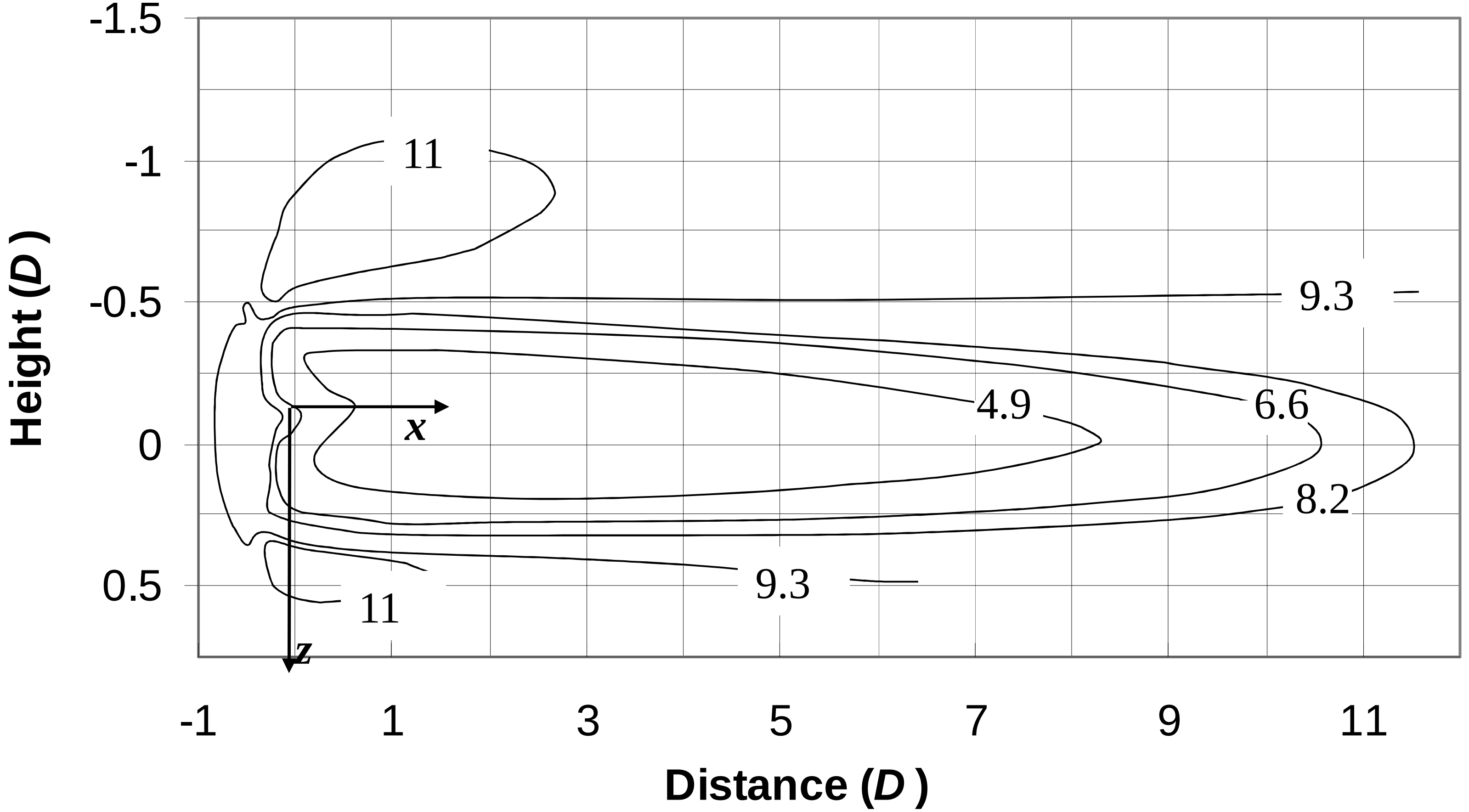
   - **The domain contained only $2x10^6$ cells.**

   - **All BC imported from the initial full domain solution were used as inlet boundary conditions for the downstream sub-domain analysis.**

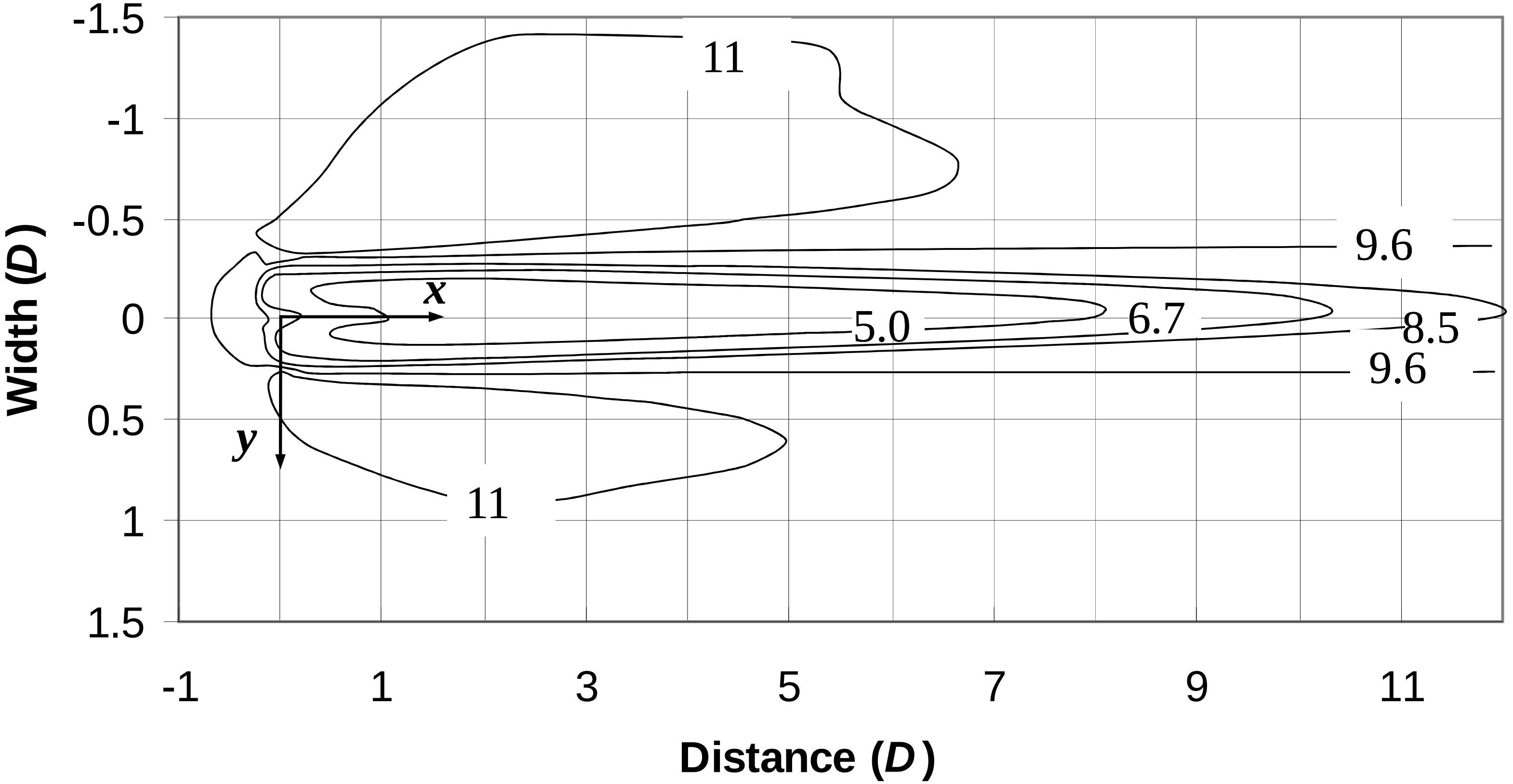
   - **Further refinement of the computational sub-domain cells divided at the steep vorticity gradient region, vorticity value 0.05 rad/sec $\leq w \leq$ 0.2 rad/sec resulting in about $5x10^6$ cells.**

A comparison between the refined and not refined solution revealed insignificant differences (no more than 5%) between the corresponding velocity deficit values $(U_0-u_x)/U_0$.

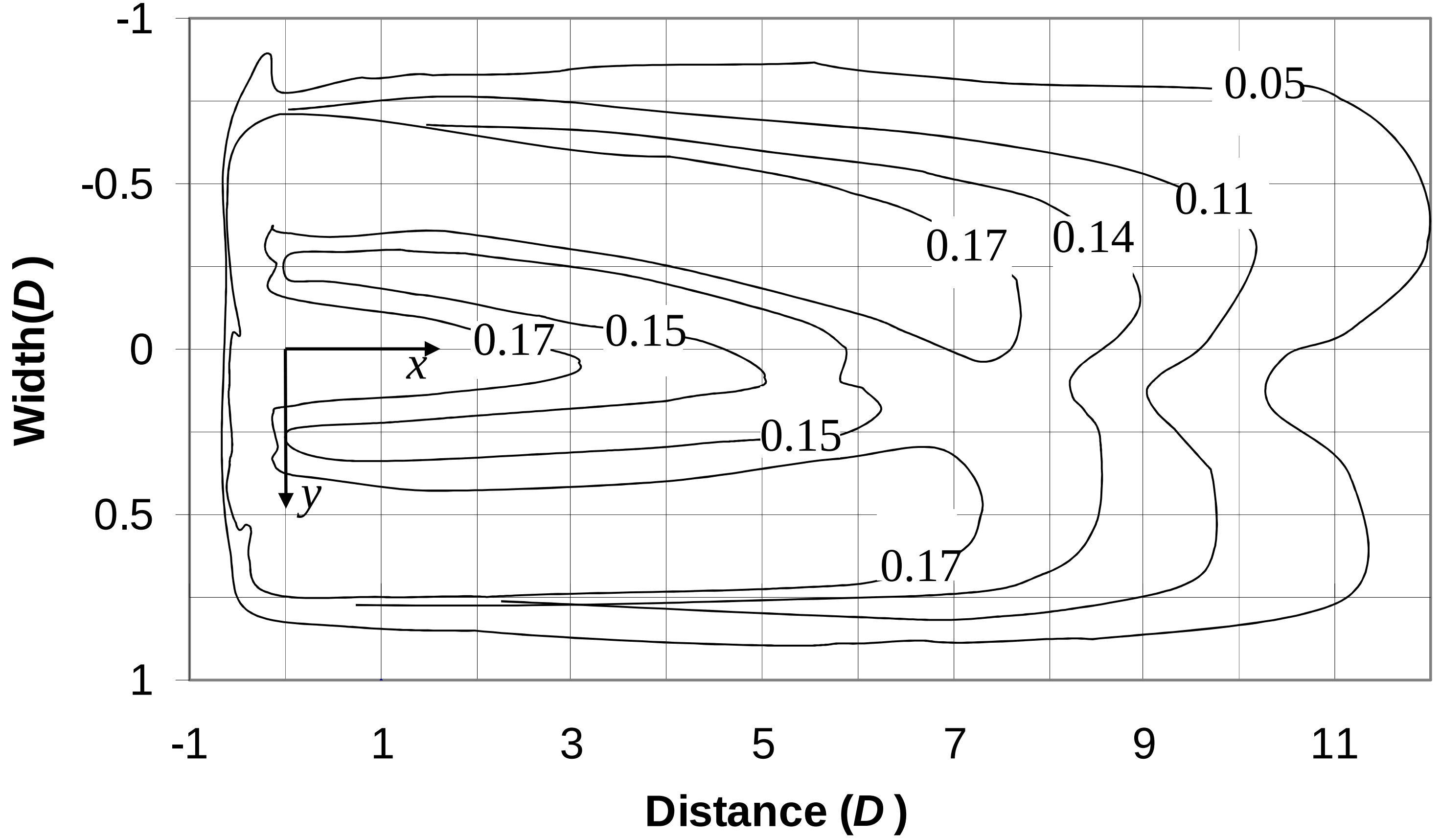
Iso-curves (m/sec) of the velocity component in x direction (the mid lengthwise vertical section)- wind 10 m/sec



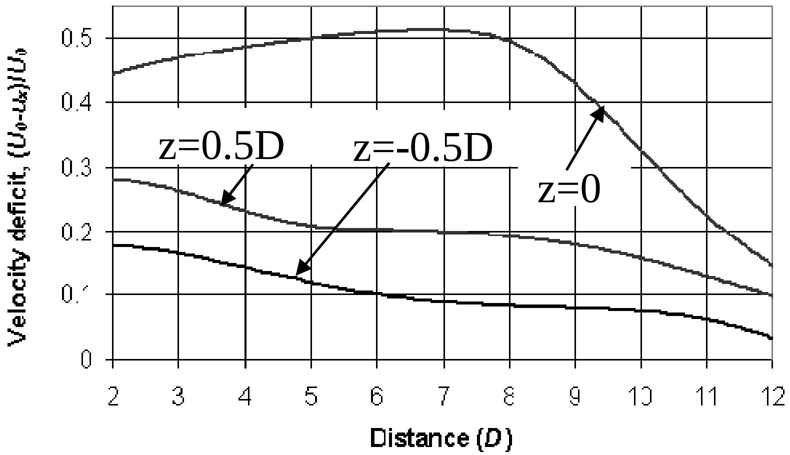
Iso-curves (m/sec) of the velocity component in x direction (the mid horizontal section)- wind 10 m/sec



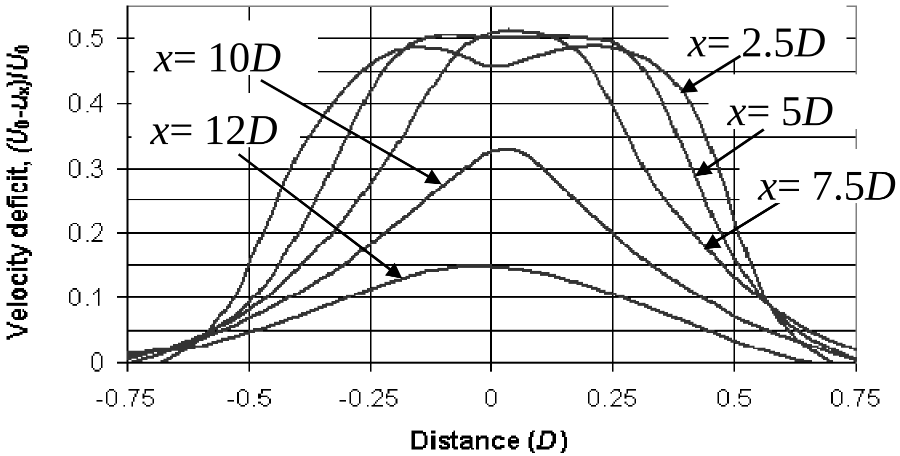
Vorticity distribution in the mid horizontal section.



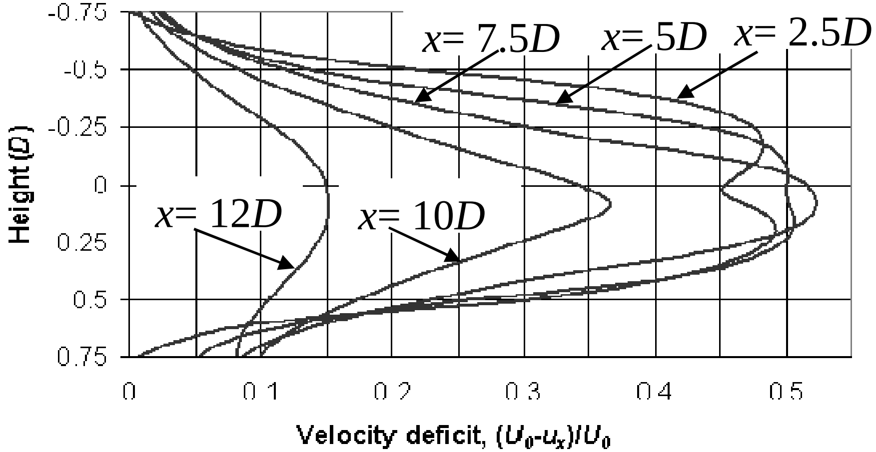
Velocity deficit distribution



in the mid lengthwise section (x-z plane)

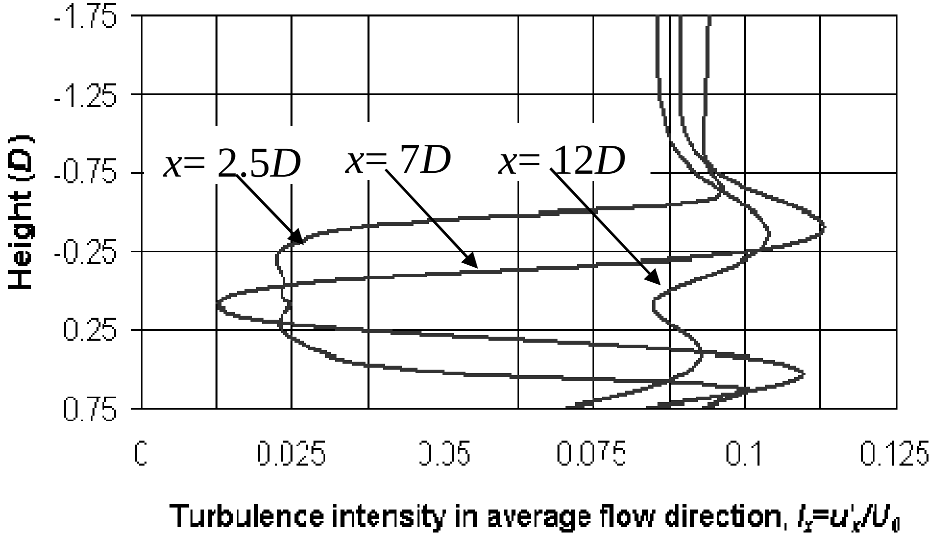


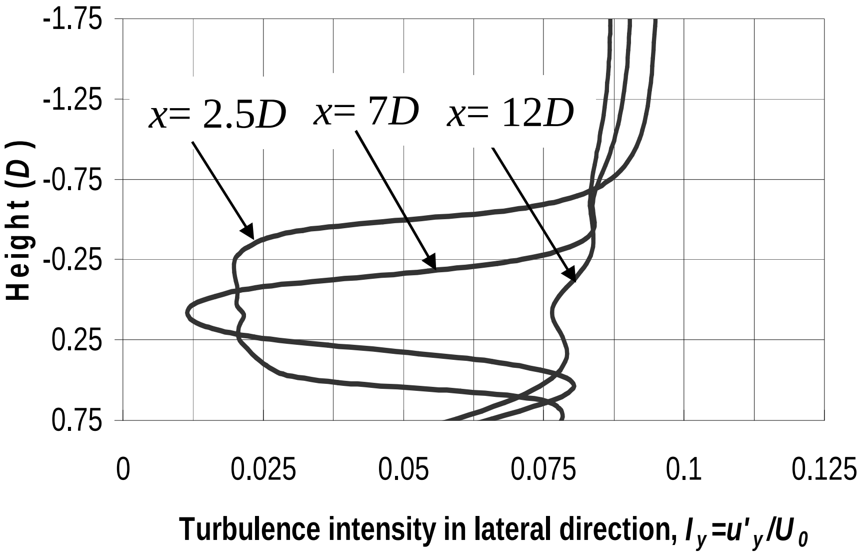
in the mid cross section (x-y plane)

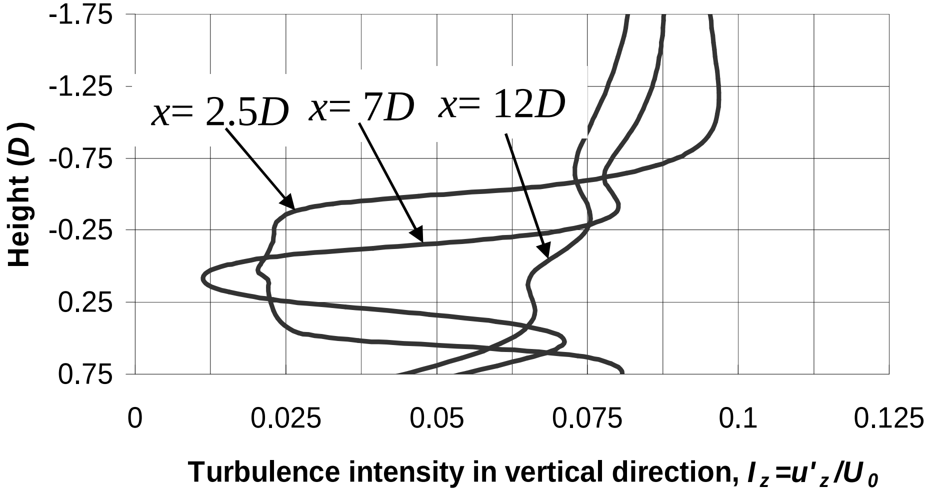


in the mid lengthwise section (x-z plane)

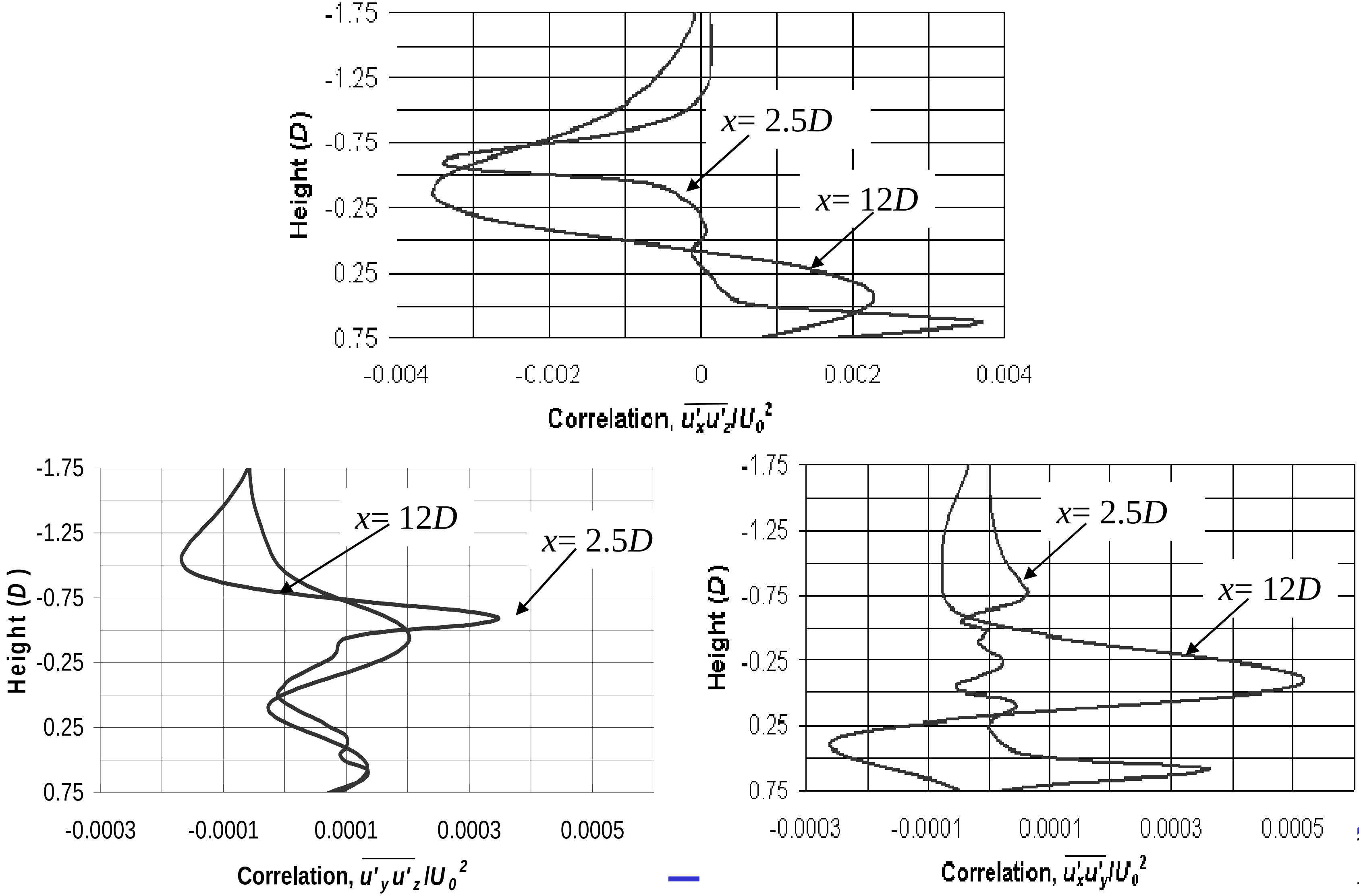
Turbulence intensity distribution in the mid lengthwise section (x-z plane) at several downstream distances:







Dimensionless correlation of fluctuating velocities in the near and far wake (x-z plane)



Conclusions

- **A full scale CFD analysis was performed to investigate the wake characteristics of a NedWind 46/3/500 turbine.**
- **The anisotropic nature of the model provides a physically relevant description of the turbulence intensity and correlations fields at any point of the computational domain.**
- **It was found that the near wake is characterized by a more isotropic behavior than the far wake. Cardinal for additional turbine location.**
- **An acceptable qualitative agreement with previous numerical and experimental studies was found.**
- **The approach does not require extensive programming and/or stringent mathematical constraints as do other works in the literature.**
- **The model implementation requires no extra features than those offered by commercial software and may be safely utilized by wind turbine engineers for a preliminary analysis.**